CS 590 Theory of Computation: Spring 1997
Homework 1

Due 3 February 1997 (Lecture 5)

Problems from the text are denoted by C or BC, depending on whether they are from the Cutland text or from the Bovet and Crescenzi text, followed by the chapter and problem number, separated by a period. For example, C 2.3.3 is problem 3 for chapter 2, section 3 of Cutland.

---

**Question 1 (C 2.3 1a)** Without writing any programs, show that for every \( m \in N \) the following functions are computable:

1. \( m \) (recall that \( m(x) = m \), for all \( x \). That is, \( m \) is the constant function equal to the number \( m \) everywhere.)

2. \( mx \) (That is, \( f(x) = mx \).)

**Question 2 (C 2.3 2)** Suppose that \( f(x, y) \) is computable, and that \( m \in N \). Show that the function \( h(x) = f(x, m) \) is computable.

**Question 3 (C 2.4 1a,b,f)** Show that the following functions are computable:

a. Any polynomial function \( \sum_{i=0}^{n} a_i x^i \), where each \( a_i \) is in \( N \).

b. \([\sqrt{x}]\). Recall that (page 22, item 1(f)! [x] is the greatest integer less than or equal to \( x \).

f. \( \tau(x) \) (this function is usually denote \( \tau(x) \) and is called Euler's function—your text denotes it \( \phi(x) \)) which is the number of positive integers less than \( x \) which are relatively prime to \( x \). (We say that \( x \) and \( y \) are relatively prime if \( HCF(x, y) = 1 \).)

**Question 4 (C 2.5 1)** Suppose that \( f(x) \) is a total injective computable function. Prove that \( f^{-1} \) (the inverse of \( f \)) is computable.

**Question 5** Thought question: where does your proof in the last question break down if \( f \) is not total? Where does it fail if \( f \) is not injective?

**Question 6** Let the \( A(n) = \psi(n, n) \), for Ackerman’s function \( \psi \) on page 46 of Cutland. Make a table for \( A(n) \) for as many values as you can stand.